

An Example Routine Calling the Mathematica Implementation of the Anderson-Moore(AIM) Algorithm

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Abstract

This paper describes how to use the Mathematica implementation of the Anderson-Moore Algorithm[1, 2, 3] for imposing the saddle point property in dynamic models. The paper uses a simple two-equation firm value model to demonstrate model construction and solution.

Contents

1	Introduction and Summary	2
2	The Firm Value Model	2
3	Model Representation and Preprocessing	2
4	Using the Anderson Moore Algorithm	3
A	Files	9
B	Macros	9
C	Identifiers	9

1 Introduction and Summary

This paper describes how to use the Mathematica implementation (currently downloadable at <http://federalreserve.gov/pubs/oss/oss4/code.html>) of the Anderson-Moore Algorithm[1, 2, 3] for imposing the saddle point property in dynamic models. The paper uses a simple two-equation model to demonstrate model construction and solution.

2 The Firm Value Model

This paper uses AIM to investigate the solution of a simple linear model, first presented in [3], which describes the value of a firm.

The model consists of two equations:

$$\begin{aligned} V_{t+1} &= (1 + r)V_t - D_{t+1} \\ D_t &= (1 - \delta)D_{t-1} \end{aligned} \quad (1)$$

where V is the value of the firm, D is the dividend, r is the interest rate, δ is the growth rate of the dividend (here, negative).

3 Model Representation and Preprocessing

Describe the linear model using the MDLEZ syntax and save the model in a file, here called firmvalue.mdl

MODEL> Provides a name for the model. This does not affect the AIM calculations.

ENDOG> Provides names of the endogenous variables. In the AIM formulation, modellers must completely describe the long run behavior of the system. As a result, all variables are endogenous. “Exogenous” variables must have, at least, a simple forecasting equation.

EQUATION> Provides a name for the equation. This does not affect the AIM calculations.

EQTYPE> Specifies the type of equation. This does not affect the AIM calculations. The keyword STOCH indicates the equation has a stochastic error term. The keyword IMPOSED indicates the equation has no error term.

EQ> Provides the model equation.

```

"firmvalue.mdl" 3 ≡

MODEL> FIRMVALUE

ENDOG>
    V
    DIV

EQUATION> VALUE
EQTYPE> IMPOSED
EQ>      LEAD(V,1) = (1+R)*V - LEAD(DIV,1)

EQUATION> DIVIDEND
EQTYPE> IMPOSED
EQ>      DIV = (1-DELTA)*LAG(DIV,1)

END

◊

```

Download the Mathematica preprocessor. Run the model file through the model preprocessor and redirect the output to an appropriate file. If working in UNIX you would type:

```
mdlezAimMath.exe firmvalue.mdl > firmvalue.math
```

This file firmvalue.math contains Mathematica code that describes the model.

4 Using the Anderson Moore Algorithm

Create a program to call AIM and the supporting routines in firmvalue.math. The small numbers to the right of the descriptions refer to sections in the paper describing the Mathematica code.

Make sure the Mathematica code read in with the Needs function is in the the current working directory, or specify the pathway in the Needs function.

```

"firmvalueMathNumeric.m" 4 ≡

Needs[ "numericLinearAim`", "numericLinearAim.m" ]
<<firmvalue.math
{modName,numEqs,numParams,numLags,numLeads}=computeAimDimensions[ "FIRMVALUE" ]
{eqNames,eqTypes,paramNames,varNames,varDelays,varTypes}=
computeAimData[ "FIRMVALUE" ]

hmat=makeHmat[ "FIRMVALUE", {0.1,0.6} ]

{{zf,hf},{zb,hb}}=numericBiDirectionalAR[hmat]

transMat=numericTransitionMatrix[hf];
{t0,j0,pi,lilTransMat}=numericSqueeze[Join[{zf, zb}], transMat];
Eigenvalues[transMat]
Eigenvalues[lilTransMat]

bles=Eigensystem[Transpose[transMat]]
ules=Eigensystem[Transpose[lilTransMat]]
mapper=numericMapper[pi,j0]

ubigEv=numericParticularLam[ules,1,mapper,t0]
ubigEvs=numericEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,t0]
◊

```

This code produces the following output:

```

In[1]:= Needs[ "numericLinearAim`", "numericLinearAim.m" ]

In[2]:= <<firmvalue.math

In[3]:= {modName,numEqs,numParams,numLags,numLeads}=computeAimDimensions[ "FIRMVA

Out[3]= {FIRMVALUE, 2, 2, 1, 1}

In[4]:= {eqNames,eqTypes,paramNames,varNames,varDelays,varTypes}=
computeAimData[ "FIRMVALUE" ]

Out[4]= {{VALUE, DIVIDEND}, {1, 1}, {R, DELTA}, {V, DIV}, {0, 0, 0},
> {0, 0, 0}}

In[5]:= hmat=makeHmat[ "FIRMVALUE", {0.1,0.6} ]

Out[5]= {{0, 0, -1.1, 0, 1, 1.}, {0, -0.4, 0, 1, 0, 0} }

In[6]:= {{zf,hf},{zb,hb}}=numericBiDirectionalAR[hmat]

```

```

Out[6]= {{{{0., -0.4, 0., 1.}}},  

>     {{0., 0., 1.1, 0., -1., -1.}, {0., 0., 0., -0.4, 0., 1.}}},  

>     {{{{1.1, 0., -1., -1.}}}, {{0., -0.4, 0., 1., 0., 0.}},  

>     {1.1, 0., -1., -1., 0., 0.}}}}  

In[7]:= transMat=numericTransitionMatrix[hf];  

In[8]:= {t0g,j0,pi,lilTransMat}=numericSqueeze[Join[zf,zb],transMat];  

In[9]:= Eigenvalues[transMat]  

Out[9]= {1.1, 0.4, 0., 0.}  

In[10]:= Eigenvalues[lilTransMat]  

Out[10]= {1.1, 0.4}  

In[11]:= bles=Eigensystem[Transpose[transMat]]  

Out[11]= {{1.1, 0.4, 0., 0.}, {{0., 0., 0.868243, -0.496139},  

>     {0., 0., 0., 1.}, {0., 0.371391, 0., -0.928477},  

>     {0.61396, 0., -0.558146, -0.558146}}}  

In[12]:= ules=Eigensystem[Transpose[lilTransMat]]  

Out[12]= {{1.1, 0.4}, {{-0.446211, -0.894928}, {-1., 1.32053 10^-17}}}  

In[13]:= mapper=numericMapper[pi,j0]  

Out[13]= Function[evMat$, Inverse[kron[IdentityMatrix[Length[{6.19666 10^-17,-3.75479 10^-17},{9.29499 10^-17,1.24471 10^-16}]], evMat$] -  

>     kron[Transpose[{{6.19666 10^-17,-3.75479 10^-17},{9.29499 10^-17,1.24471 10^-16}}]], IdentityMatrix[Length[evMat$]]]]\
```

```

>      . kron[Transpose[{{-0.619617, -0.829743}, {0.664545, -0.723993}}], 
>      IdentityMatrix[Length[evMat$]]]

In[14]:= ubigEv=numericParticularLam[ules,1,mapper,tog]

          -16          -17
Out[14]= {{1.11022 10^-, -5.55112 10^-, -1.20946, 0.69112} }

In[15]:= ubigEvs=numericEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,tog]

          -16          -17
Out[15]= {{1.11022 10^-, -5.55112 10^-, -1.20946, 0.69112}, 

          -16          -16          -16
>     {-2.44238 10^-, -4.44089 10^-, 5.14045 10^-, 2.77534} }

"firmvalueMathSymbolic.m" 6 ≡

Needs[ "symbolicLinearAim` ", "symbolicLinearAim.m" ]
<<firmvalue.math
{modName,numEqs,numParams,numLags,numLeads}=computeAimDimensions[ "FIRMV
{eqNames,eqTypes,paramNames,varNames,varDelays,varTypes}=computeAimData[ "FIRMV
hmatS=makeHmat[ "FIRMV", {r,delta}]

{{zf,hf},{zb,hb}}=symbolicBiDirectionalAR[hmatS]

{pm,j0,pi,lilTransMat}=symbolicSqueeze[Join[zf,zb],symbolicTransitionMatrix[hf]];
bles=Eigensystem[Transpose[transMat]];
ules=Eigensystem[Transpose[lilTransMat]];
mapper=symbolicMapper[pi,j0];
ubigEv=symbolicParticularLam[ules,1,mapper,pm];

subs={r->0.1,delta->0.6};
ubigEvs=symbolicEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,pm];
ubigEvs/.subs
◊

```

This code produces the following output:

```

In[1]:= Needs[ "symbolicLinearAim` ", "symbolicLinearAim.m" ]

In[2]:= <<firmvalue.math

```

```

In[3]:= {modName,numEqs,numParams,numLags,numLeads}=computeAimDimensions["FIRMVA"]

Out[3]= {FIRMVALUE, 2, 2, 1, 1}

In[4]:= {eqNames,eqTypes,paramNames,varNames,varDelays,varTypes}=
computeAimData["FIRMVALUE"]

Out[4]= {{VALUE, DIVIDEND}, {1, 1}, {R, DELTA}, {V, DIV}, {0, 0, 0},
> {0, 0, 0} }

In[5]:= hmatS=makeHmat["FIRMVALUE",{r,delta}]

Out[5]= {{0, 0, -1. - r, 0, 1, 1.}, {0, -1. + 1. delta, 0, 1, 0, 0} }

In[6]:= {{zf,hf},{zb,hb}}=symbolicBiDirectionalAR[hmatS]

Out[6]= {{{{0., 1. (-1. + 1. delta), 0, 1.}},
> {0., 0. + 0. (-1. + 1. delta), 0. + 1. (-1. - r),
> 0. - 1. (-1. + 1. delta), 1., 0.},
> {0., 0. + 0. (-1. + 1. delta), 0. + 0. (-1. - r),
> 0. + 1. (-1. + 1. delta), 0., 1.}}},
> {{{-1. - r, 0, 1, 1.}}, {{0, 1, 0, 1, 0, 0},
> -1. + 1. delta
> {1, 0, 1, 0, 1, 0}}}}

```

```
In[12]:= subs={r->0.1,delta->0.6};

In[13]:= ubigEvs=symbolicEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,pm]

In[14]:= ubigEvs/.subs

Out[14]= {{0., 0., 0., 1.}, {0., 0., -1.75, 1.}}
```

A Files

"firmvalue.mdl" Defined by scrap 3.
"firmvalueMathNumeric.m" Defined by scrap 4.
"firmvalueMathSymbolic.m" Defined by scrap 6.

B Macros

C Identifiers

References

- [1] Gary Anderson. A reliable and computationally efficient algorithm for imposing the saddle point property in dynamic models. Unpublished Manuscript, Board of Governors of the Federal Reserve System. Downloadable copies of this and other related papers at <http://irmum1.frb.gov/~m1gsa00/summariesAbstracts.html>, 1997.
- [2] Gary Anderson and George Moore. An efficient procedure for solving linear perfect foresight models. Unpublished Manuscript, Board of Governors of the Federal Reserve System. Downloadable copies of this and other related papers at <http://irmum1.frb.gov/~m1gsa00/summariesAbstracts.html>, 1983.
- [3] Gary Anderson and George Moore. A linear algebraic procedure for solving linear perfect foresight models. *Economics Letters*, 17, 1985.
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- [9] Ben Noble. *Applied Linear Algebra*. Prentice-Hall, Inc., 1969.
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